Review of the FoPL paper [1]

The Evans Lemma of Differential Geometry

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Abstract

The Evans Lemma is a basic tool for Evans GCUFT or ECE Theory [2]. Evans has given two proofs of his Lemma. The first proof in [1] is shown to be invalid due to dubious use of the covariant derivative D^μ. A second proof in [2, Sec.J.3] is wrong due to a logical error.

1 M.W. Evans' first proof of his Lemma

The Evans Lemma is the assertion of proportionality of the matrices $(\Box q^a_\mu)$ and (q^a_μ) with a proportionality factor R:

$$(\Box q^a_\mu) = R(q^a_\mu).$$

Quotation from [1, p.432+8][1]

The Evans lemma is a direct consequence of the tetrad postulate. The proof of the lemma starts from covariant differentiation of the postulate:

[1, (36)][2] $$D^\mu(\partial_\mu q^a_\lambda + \omega^a_{\mu b} q^b_\lambda - \Gamma^\nu_{\mu\lambda} q^a_\nu) = 0.$$

Using the Leibnitz rule, we have

[1, (37)] $$(D^\mu \partial_\mu) q^a_\lambda + \partial_\mu(D^\mu q^a_\lambda) + (D^\mu \omega^a_{\mu b}) q^b_\lambda + \omega^a_{\mu b}(D^\mu q^b_\lambda) - (D^\mu \Gamma^\nu_{\mu\lambda}) q^a_\nu - \Gamma^\nu_{\mu\lambda}(D^\mu q^a_\nu) = 0,$$

and so

[1, (38)] $$(D^\mu \partial_\mu) q^a_\lambda + (D^\mu \omega^a_{\mu b}) q^b_\lambda - (D^\mu \Gamma^\nu_{\mu\lambda}) q^a_\nu = 0,$$

because

[1, (39)] $$D^\mu q^a_\lambda = D^\mu q^b_\lambda = D^\mu q^a_\nu = 0.$$

End of Quotation

Eq.[1, (36)] is formally correct, however, the decomposition in Eq.[1, (37)] yields *undefined* expressions: What e.g. is the meaning of the terms $D^\mu \omega^a_{\mu b}$ and $D^\mu \Gamma^\nu_{\mu\lambda}$? Note that both $\omega^a_{\mu b}$ and $\Gamma^\nu_{\mu\lambda}$ are no tensors and so the covariant derivative D^μ is not applicable. Therefore we skip over the rest of [1].

2 Evans' second proof of his Lemma

M.W. Evans himself felt it necessary to give another proof in [2, p.514],

[1] The page numbers of the web copy mentioned in [1] start with 1 instead of 433 (= 432+1).

[2] Quotations from M.W. Evans' contributions [1], [2] and [3] appear with equation labels [p,(nn)] in the left margin.

now avoiding the problem of undefined terms.

Quotation from [2, p.514]

J.3 The Evans Lemma

The Evans Lemma is the direct result of the tetrad postulate of differential geometry:

[2, (J.27)] $$D_\mu q^a_\lambda = \partial_\mu q^a_\lambda + \omega^a_{\mu b} q^b_\lambda - \Gamma^\nu_{\mu\lambda} q^a_\nu = 0.$$

using the notation of the text. It follows from eqn. (J.27) that:

[2, (J.28)] $$D^\mu(D_\mu q^a_\lambda) = \partial^\mu(D_\mu q^a_\lambda) = 0,$$

i.e.

[2, (J.29)] $$\partial^\mu(\partial_\mu q^a_\lambda + \omega^a_{\mu b} q^b_\lambda - \Gamma^\nu_{\mu\lambda} q^a_\nu) = 0,$$

or

[2, (J.30)] $$\Box q^a_\lambda = \partial^\mu(\Gamma^\nu_{\mu\lambda} q^a_\nu) - \partial^\mu(\omega^a_{\mu b} q^b_\lambda).$$

Define:

[2, (J.31)] $$R q^a_\lambda := \partial^\mu(\Gamma^\nu_{\mu\lambda} q^a_\nu) - \partial^\mu(\omega^a_{\mu b} q^b_\lambda)$$

to obtain the Evans Lemma:

[2, (J.32)] $$\Box q^a_\lambda = R q^a_\lambda$$

End of Quotation

As simple as wrong: Eq.[2, (J.31)] represents a set of 16 equations each of which for one fixed pair of indices $(a, \mu)(a, \mu = 0, 1, 2, 3)$. Each equation is a condition to be fulfilled by the quantity R. These 16 conditions for R do *not agree* in general.

Thus, the author Evans, when giving the "definition" [2, (J.31)], ignored the possible incompatibility of the *sixteen* definitions of R contained in his "definition" of R by Eq.[2, (J.31)]. Therefore this proof of the Evans Lemma in [2, Sec.J.3] is invalid.

Conclusion: There is no proof of the Evans Lemma, neither in the article [1] nor in [2, Sec.J.3].

Additional remark In his note [3, p.2] Evans gives a variation of this "proof". There he defines R directly and applies his "Cartan Convention":

Quotation from [3]

[3, (9)] $$R = q_a^\lambda \partial^\mu (\Gamma^\nu_\mu \lambda q_\nu^a - \omega^a_{\mu b} q_\lambda^b)$$

and use <the "Cartan Convention">

[3, (10)] $$q_a^\lambda q_\lambda^a = 1$$

to find

[3, (11)] $$\Box q_\lambda^a = R q_\lambda^a.$$

End of Quotation

i.e. from the correct Eq. [2,(J.30)] he *erroneously* concludes

$$q_\lambda^a R = (q_\lambda^a q_a^\lambda) \partial^\mu (\Gamma^\nu_\mu \lambda q_\nu^a - \omega^a_{\mu b} q_\lambda^b) = 1 \cdot \Box q_\lambda^a$$

We learn from this that one can "prove" every nonsense, if one has the suitable error at hand, e.g. ignore the rules of tensor calculus on hidden indices. (see also [5, Evans' New Math in Full Action ...])

References

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